

a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

b) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$

c) $\left(\frac{x_1-y_1}{2}, \frac{x_2-y_2}{2}\right)$

d) $\left(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}\right)$

13. If the arithmetic mean of 7, 8, x, 11, 14 is x, then x = [1]

a) 10

b) 10.5

c) 9.5

d) 9

14. If $2 \cos 3\theta = 1$ then $\theta = ?$ [1]

a) 30°

b) 10°

c) 15°

d) 20°

15. In a right ΔXYZ , XZ is the hypotenuse of length 12 cm and $\angle X = 45^\circ$. The area of the triangle is [1]

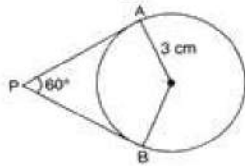
a) 72cm^2

b) 12cm^2

c) 24cm^2

d) 36cm^2

16. If two tangents inclined at 60° are drawn to circle of radius 3 cm, then length of each tangent is equal to [1]



a) $3\sqrt{3}$

b) 3 cm

c) $2\sqrt{3}$ cm

d) $3\sqrt{2}$ cm

17. D and E are respectively the points on the sides AB and AC of a triangle ABC such that AD = 2 cm, BD = 3 cm, BC = 7.5 cm and $DE \parallel BC$. Then, length of DE (in cm) is [1]

a) 6

b) 5

c) 2.5

d) 3

18. Which of the given is a quadratic equation? [1]

a) $x + \frac{1}{x} = x^2$

b) $x + \frac{1}{x^2} = 5$

c) $x^2 - 3\sqrt{x} + 2 = 0$

d) $2x^2 - 5x = (x - 1)^2$

19. **Assertion (A):** If the sum of the zeroes of the quadratic polynomial $x^2 - 2kx + 8$ are is 2 then value of k is 1. [1]

Reason (R): Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $-\frac{b}{a}$

- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2 . [1]

Reason (R): Total surface area of a cuboid is $2(lb + bh + hl)$

- a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false. d) A is false but R is true.

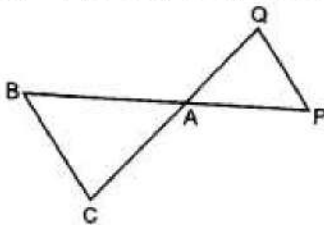
Section B

21. Find the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$. If the real roots exist. Find it. [2]
22. Determine, by distance formula, whether the given points are collinear : (1, 2), (5, 3) and (18, 6). [2]

OR

Do the points (3, 2), (-2, -3) and (2, 3) form a triangle? If so, name the type of triangle formed.

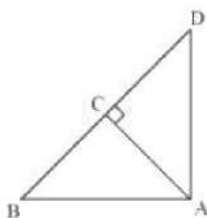
23. An electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 a.m. At what time will they beep together at the earliest? [2]
24. Prove that: $\sin \theta \cos(90^\circ - \theta) + \cos \theta \cdot \sin(90^\circ - \theta) = 1$ [2]
25. In the given figure, $\triangle ACB \sim \triangle AQP$. If $BC = 8 \text{ cm}$, $PQ = 4 \text{ cm}$, $BA = 6.5 \text{ cm}$, $AQ = 2.8 \text{ cm}$, find CA and PA. [2]



OR

$\triangle ABD$ is a right triangle right-angled at A and $AC \perp BD$. Show that $AB^2 = BC \times BD$





Section C

26. Find three consecutive positive integers whose product is equal to sixteen times their sum. [3]
27. ABCD is a rectangle in which $BC = 2AB$. A point E lies on ray CD such that $CE = 2BC$. Prove that $BE \perp AC$. [3]
28. Find the ratio in which the point $(x, 2)$ divides the line segment joining the points $(-3, -4)$ and $(3, 5)$. Also find the value of x . [3]

OR

Find the values of x for which the distance between the point P $(2, -3)$ and Q $(x, 5)$ is 10.

29. In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps? [3]
30. The angle of elevation of an aeroplane from a point on the ground is 45° . After flying for 15 s, the angle of elevation changes to 30° . If the aeroplane is flying at a constant height of 2500 m, then find the average speed of the aeroplane. [3]

OR

From the top of a building AB, 60m high, the angles of depression of the top and bottom of a vertical lamp-post CD are observed to be 30° and 60° respectively. Find the difference between the heights of the building and the lamp-post.

31. The median of the following data is 52.5. Find the values of x and y if the total frequency is 100. [3]

Class	frequency
0 - 10	2
10 - 20	5
20 - 30	x
30 - 40	12
40 - 50	17
50 - 60	20
60 - 70	y
70 - 80	9
80 - 90	7

Section D

32. DDA wants to make a rectangular park in the colony. If the length and breadth of the park are decreased by 2 m, then the area will be decreased by 196 sq meters. Its area will be increased by 246 sq meters if its length is increased by 3 m and breadth is increased by 2 m. Find the length and breadth of the park. [5]

OR

A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

33. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$. [5]
34. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre. [5]

OR

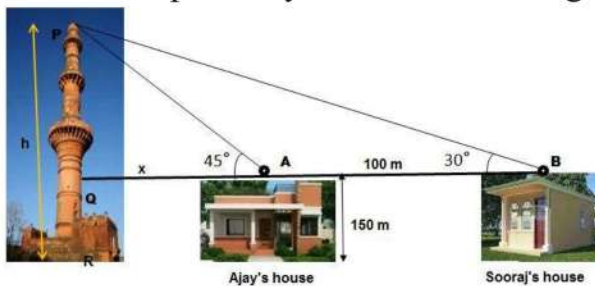
Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use $\pi = 22/7$).

35. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is (i) a king or jack (ii) a non-ace (iii) a red card (iv) neither a king nor a queen. [5]

Section E

36. **Read the text carefully and answer the questions:** [4]

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45° and 30° respectively as shown in the figure.



- (i) Find the height of the tower.
- (ii) What is the distance between the tower and the house of Sooraj?
- (iii) Find the distance between top of the tower and top of Sooraj's house?



OR

Find the distance between top of tower and top of Ajay's house?

37. **Read the text carefully and answer the questions:**

[4]

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



- (i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it
- (ii) Find the total money he saved.
- (iii) How much money Akshar saves in 10 days?

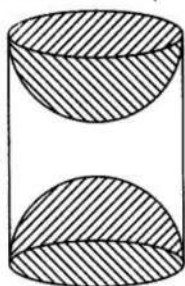
OR

How many coins are there in piggy bank on 15th day?

38. **Read the text carefully and answer the questions:**

[4]

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



- (i) Find the volume of the cylindrical block before the carpenter started scooping the hemisphere from it.
- (ii) Find the volume of wood scooped out?
- (iii) Find the total surface area of the article?

OR



Find the total surface area of cylinder before scooping out hemisphere?

Solution

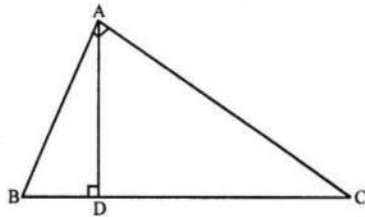
Section A

1. (d) $\left(\frac{AB}{AC}\right)^2$

Explanation: In right angled $\triangle ABC$, $\angle A = 90^\circ$

$AD \perp BC$

$\therefore \triangle ABD \sim \triangle ABC$



$$\frac{AB}{BC} = \frac{BD}{AB} \Rightarrow AB^2 = BD \times BC \dots\dots(i)$$

Similarly $\triangle ACD \sim \triangle ABC$

$$DC \times BC = AC^2 \dots\dots\dots(ii)$$

Dividing (ii) by (i)

$$\frac{BD \times BC}{DC \times BC} = \frac{AB^2}{AC^2} \Rightarrow \frac{BD}{DC} = \frac{AB^2}{AC^2}$$

Hence $\frac{BD}{DC} = \frac{AB^2}{AC^2}$

2. (c) $3\sqrt{2}, -2\sqrt{2}$

Explanation: $x^2 - \sqrt{2}x - 12 = x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12$

$$= x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2}) = (x - 3\sqrt{2})(x + 2\sqrt{2})$$

$$\therefore x = 3\sqrt{2} \text{ or } x = -2\sqrt{2}$$

3. (a) unique solution

Explanation: $2x + 3y - 7 = 0$

$$6x + 5y - 11 = 0$$

By Comparing with $a_1x + b_1y + c = 0$ and $a_2x + b_2y + c = 0$,

Here, $a_1 = 2, b_1 = 3, c_1 = -7$, and $a_2 = 6, b_2 = 5, c_2 = -11$

$$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{3}{5}$$



Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, the system of equations has a unique solution.

4. (c) $k = 3$

Explanation: Given: $a_1 = k, a_2 = 6, b_1 = -1, b_2 = -2, c_1 = -2$ and $c_2 = -3$

If there is no solution, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{k}{6} = \frac{-1}{-2} \neq \frac{-2}{-3} \text{ Taking } \frac{k}{6} = \frac{-1}{-2}$$

$$\Rightarrow k = \frac{6}{2} \Rightarrow k = 3$$

5. (a) 2.1 cm

Explanation: In $\triangle ABC$, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{3}{5}, AC = 5.6 \text{ cm}$$

Let $AE = x$ cm, the $EC = 5.6 - x$

In $\triangle ABC$, $DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AE}{EC} = \frac{3}{5} \Rightarrow \frac{x}{5.6-x} = \frac{3}{5}$$

$$\Rightarrow 5x = 16.8 - 3x$$

$$\Rightarrow 5x + 3x = 16.8 \Rightarrow 8x = 16.8$$

$$\Rightarrow x = \frac{16.8}{8} = 2.1$$

$$\therefore x = 2.1 \text{ cm}$$

6. (a) $\frac{1}{6}$

Explanation: No. of months in a year = 12

$$\text{Probability of being March or October} = \frac{2}{12} = \frac{1}{6}$$

7. (b) 9

Explanation: $9(\sec^2 A - \tan^2 A)$

$$= 9 \times 1 (\sec^2 A - \tan^2 A = 1)$$

$$= 9$$

8. (d) 40

Explanation: Arranging the given data in ascending order: 23, 27, 29, 30, 40, 44, 46, 52, 59

Here $n = 9$, which is odd.

$$\therefore \text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}}$$

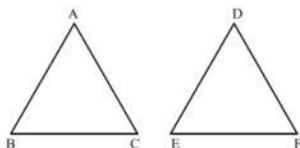
$$= \left(\frac{9+1}{2}\right)^{\text{th}} \text{ term}$$

$$= 5^{\text{th}} \text{ term}$$

$$= 40$$

9. (d) 50°

Explanation:



$$\triangle ABC \sim \triangle DEF$$

$$\angle A = 47^{\circ}, \angle E = 83^{\circ}$$

$\triangle ABC$ and $\triangle DEF$ are similar

$$\angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F$$

$$\angle A = 47^{\circ}$$

$$\angle B = \angle E = 83^{\circ}$$

But $\angle A + \angle B + \angle C = 180^{\circ}$ (Sum of angles of a triangle)

$$47^{\circ} + 83^{\circ} + \angle C = 180^{\circ}$$

$$\Rightarrow 130^{\circ} + \angle C = 180^{\circ}$$

$$\Rightarrow \angle C = 180^{\circ} - 130^{\circ}$$

$$\Rightarrow \angle C = 50^{\circ}$$

10. (c) 180

Explanation: It is given that: $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$

\therefore HCF (a, b) = Product of smallest power of each common prime factor in the numbers

$$= 2^2 \times 3^2 \times 5 = 180$$

11. (c) $\frac{-8}{5} < k < \frac{8}{5}$

Explanation: For no real roots, we must have $b^2 - 4ac < 0$.

$$\therefore (25k^2 - 4 \times 16) < 0 \Rightarrow 25k^2 < 64 \Rightarrow k^2 < \frac{64}{25} \Rightarrow \frac{-8}{5} < k < \frac{8}{5}$$

12. (a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Explanation: we know that the midpoint formula = $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and $(x_2,$

$y_2)$ is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

13. (a) 10

Explanation: Arithmetic mean of 7, 8, x, 11, 14 is x

$$\Rightarrow \frac{7+8+x+11+14}{5} = x$$

$$\Rightarrow \frac{40+x}{5} = x \Rightarrow 40+x=5x$$

$$\Rightarrow 5x-x=40 \Rightarrow 4x=40$$

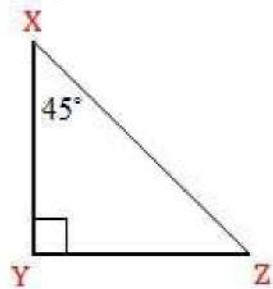
$$\Rightarrow x = \frac{40}{4} = 10$$

14. (d) 20°

Explanation: $2\cos 3\theta = 1 \Rightarrow \cos 3\theta = \frac{1}{2} = \cos 60^\circ \Rightarrow 3\theta = 60^\circ \Rightarrow \theta = 20^\circ$

15. (d) 36cm^2

Explanation: In triangle XYZ,



$$\cos 45^\circ = \frac{XY}{XZ} \Rightarrow \frac{1}{\sqrt{2}} = \frac{XY}{12}$$

$$\Rightarrow XY = \frac{12}{\sqrt{2}} \text{ cm and } \sin 45^\circ = \frac{YZ}{XZ}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{YZ}{12}$$

$$\Rightarrow YZ = \frac{12}{\sqrt{2}} \text{ cm}$$

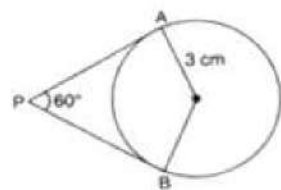
$$\therefore \text{ar}(\triangle XYZ) = \frac{1}{2} \times \frac{12}{\sqrt{2}} \times \frac{12}{\sqrt{2}}$$

$$= 36\text{cm}^2$$

16. (a) $3\sqrt{3}$

Explanation:

Let O be the centre. Construction: Joined OP.



Since OP bisects $\angle P$, therefore, $\angle APO = \angle OPB = 30^\circ$ And $\angle OAP = 90^\circ$

$$\therefore \tan 30^\circ = \frac{OA}{AP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP}$$

$\Rightarrow AP = 3\sqrt{3}$ cm Since each tangent from an external point to a circle are equal.

Therefore, $PA = PB = 3\sqrt{3}$ cm

17. (b) 5

Explanation: In $\triangle ADE$ and $\triangle ABC$

angle A common

angle D=B angle ($DE \parallel BC$ then, $d = b$)

by AA similarity criteria

$\triangle ADE$ similar $\triangle ABC$.

$$\frac{AD}{DB} = \frac{DE}{BC}$$

$$\frac{2}{3} = \frac{DE}{7.5}$$

$DE = 5$ cm.

18. (d) $2x^2 - 5x = (x - 1)^2$

Explanation: $2x^2 - 5x = (x - 1)^2$ using $(a - b)^2 = a^2 + b^2 - 2ab$

$$2x^2 - 5x = x^2 - 2x + 1$$

$$2x^2 - 5x - x^2 + 2x - 1 = 0$$

$$x^2 - 3x - 1 = 0$$

$$a = 1, b = -3 \text{ and } c = -1$$

This is of the form $ax^2 + bx + c = 0$ i.e. of degree 2 ($a \neq 0$, a, b, c are real numbers)

Hence this is a quadratic equation.

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Relation is true as we know that Sum of zeroes = $-\frac{b}{a}$

$$\Rightarrow \frac{-(-2k)}{1} = 2 \Rightarrow k = 1$$

So, Assertion is true.

20. (d) A is false but R is true.

Explanation: A is false but R is true.

Section B

21. The given quadratic equation is

$$2x^2 - 6x + 3 = 0$$

Here, $a = 2$, $b = -6$, $c = 3$

Therefore, discriminant = $b^2 - 4ac$

$$= (-6)^2 - 4(2)(3) = 36 - 24$$

$$= 12 > 0$$

So, the given quadratic equation has two distinct real roots

Solving the quadratic equation $2x^2 - 6x + 3 = 0$, by the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{we get} = \frac{-(-6) \pm \sqrt{12}}{2(2)} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Therefore, the roots are $\frac{3 \pm \sqrt{3}}{2}$, i.e. $\frac{3 + \sqrt{3}}{2}$ and $\frac{3 - \sqrt{3}}{2}$

22. Let the given points (1, 2), (5, 3) and (18, 6) be denoted by A, B, and C respectively.

$$\text{Now } AB = \sqrt{(5-1)^2 + (3-2)^2} = \sqrt{16+1} = \sqrt{17}$$

$$BC = \sqrt{(18-5)^2 + (6-3)^2} = \sqrt{169+9} = \sqrt{178}$$

$$AC = \sqrt{(18-1)^2 + (6-2)^2} = \sqrt{289+16} = \sqrt{305}$$

Here, we see that $AB + BC \neq AC$, $BC + AC \neq AB$ and $AB + AC \neq BC$. Hence, the points A, B and C are not collinear.

OR

Let us apply the distance formula to find the distances PQ, QR and PR, where

P ↔ (3, 2),

Q ↔ (-2, -3) and

R ↔ (2, 3)

are the given points. We have

$$PQ = \sqrt{(3+2)^2 + (2+3)^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 7.07(\text{ approx. })$$

$$QR = \sqrt{(-2-2)^2 + (-3-3)^2} = \sqrt{(-4)^2 + (-6)^2} = \sqrt{52} = 7.21(\text{ approx. })$$

$$PR = \sqrt{(3-2)^2 + (2-3)^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2} = 1.41(\text{ approx. })$$

Since the sum of any two of these distances is greater than the third distance, therefore, the points P, Q and R form a triangle.

Also, $PQ^2 + PR^2 = QR^2$, by the converse of Pythagoras theorem, we have $\angle P = 90^\circ$.

Therefore, PQR is a right triangle.

23. L.C.M. of 60 and 62 seconds is 1860 seconds

$$\frac{1860}{60} = 31 \text{ minutes}$$

They will beep together at 10:31 a.m.

$$24. \text{L. HS.} = \sin\theta \cdot \cos(90^\circ - \theta) + \cos\theta \cdot \sin(90^\circ - \theta)$$

$$= \sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta$$

$$= \sin^2\theta + \cos^2\theta = 1 = \text{R. HS}$$

25. According to question it is given that

$$\triangle ACB \sim \triangle AQP$$

Also, AQ = 2.8 cm, BC = 8 cm, PQ = 4 cm

$$\therefore \frac{AC}{AQ} = \frac{BC}{PQ} = \frac{AB}{AP}$$

$$\Rightarrow \frac{AC}{AQ} = \frac{BC}{PQ}$$

$$\Rightarrow \frac{AC}{2.8} = \frac{8}{4}$$

$$\Rightarrow AC = 5.6 \text{ cm}$$

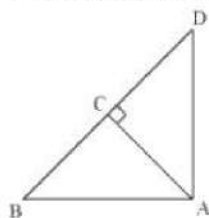
$$\text{Also, } \frac{BC}{PQ} = \frac{AB}{AP} \Rightarrow \frac{8}{4} = \frac{6.5}{AP}$$

$$\Rightarrow AP = \frac{6.5}{2} = 3.25 \text{ cm}$$

OR

Given: $\triangle ABD$ is a right triangle right-angled at A and $AC \perp BD$.

To Prove: $AB^2 = BC \times BD$



Proof: In $\triangle ADB$ and $\triangle CAB$

$$\angle DAB = \angle ACB = 90^\circ$$

$$\angle ABD = \angle CBA \text{ [common angle]}$$

$$\angle ADB = \angle CAB \text{ [remaining angle]}$$

So, $\triangle ADB \sim \triangle CAB$ (by AAA similarity)

$$\text{Therefore } \frac{AB}{CB} = \frac{BD}{AB}$$

$$\Rightarrow AB^2 = CB \times BD$$

Section C

26. Let three consecutive positive integers be x , $x + 1$ and $x + 2$

As per given condition product of three consecutive positive integers is equal to sixteen times their sum.

$$\therefore (x)(x+1)(x+2) = 16(x+x+1+x+2)$$

$$\Rightarrow (x^2+x)(x+2) = 16(3x+3)$$

$$\Rightarrow x^3 + 2x^2 + x^2 + 2x = 48x + 48$$

$$\Rightarrow x^3 + 3x^2 - 46x - 48 = 0$$

When $x = -1$, we have

$$\text{LHS} = (-1)^3 + 3(-1)^2 - 46(-1) - 48$$

$$= -1 + 3 + 46 - 48 = 0$$

$\therefore (x + 1)$ is a factor

For other factors



$$\begin{array}{r}
 x^2 + 2x - 48 \\
 x + 1 \overline{) x^3 + 3x^2 - 46x - 48} \\
 \underline{-x^3 + x^2} \\
 2x^2 - 46x \\
 \underline{2x^2 + 2x} \\
 -48x - 48 \\
 \underline{-48x - 48} \\
 + + \\
 \hline
 0
 \end{array}$$

$$\therefore x^3 + 3x^2 - 46x - 48 = 0$$

$$\Rightarrow (x + 1)(x^2 + 2x - 48) = 0$$

$$\Rightarrow (x + 1)(x + 8)(x - 6) = 0$$

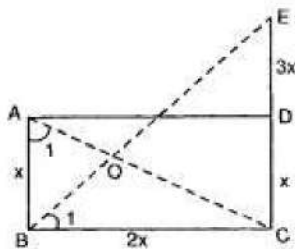
$$\Rightarrow x = -1, x = -8, x = 6.$$

Rejecting -ve values, therefore, $x = 6$

\therefore Positive integers are 6, 7 and 8.

27. To prove: $BE \perp AC$

Construction: Join AC and BE, which intersect O.



Proof: Considering $\triangle ABC$ and $\triangle BCE$,

$$\frac{AB}{BC} = \frac{BC}{CE}$$

Also, $\angle ABC = \angle BCE = (90^\circ \text{ each})$

$\therefore \triangle ABC \sim \triangle BCE$ (SAS similarity)

$$\Rightarrow \angle BAC = \angle CBE = \angle 1$$

$$\Rightarrow \angle OBA = 90^\circ - \angle 1$$

Now in $\triangle AOB$,

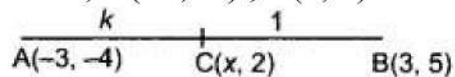
$$\angle AOB = 180^\circ - \angle OAB - \angle OBA$$

$$= 180^\circ - \angle 1 - (90^\circ - \angle 1)$$

$$= 180^\circ - \angle 1 - 90^\circ + \angle 1 = 90^\circ$$

Since $\angle AOB$ is $90^\circ \Rightarrow BE \perp AC$.

28. Given, $A(-3, -4)$, $B(3, 5)$ and $C(x, 2)$



Let C divides AB in the ratio $k : 1$

By using section formula, we get

$$(x, 2) = \left(\frac{(k \times 3) + (1 \times -3)}{k + 1}, \frac{(k \times 5) + (1 \times -4)}{k + 1} \right)$$

$$\therefore y \text{ coordinate of } C = \frac{(k \times 5) + (1 \times -4)}{k + 1}$$

$$\Rightarrow 2 = \frac{5k - 4}{k + 1}$$

$$\Rightarrow 2k + 2 = 5k - 4$$

$$\Rightarrow k = 2$$

\therefore C divides AB in the ratio 2 : 1

$$\therefore \text{ x coordinates of C} = \frac{(2 \times 3) + (1 \times -3)}{2+1}$$

$$\Rightarrow x = 1$$

OR

Distance between P(2, -3) and Q(x, 5) = 10

$$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \sqrt{(x - 2)^2 + (5 + 3)^2} = 10$$

squaring,

$$\Rightarrow (x - 2)^2 + (8)^2 = (10)^2$$

$$\Rightarrow x^2 - 4x + 4 + 64 = 100$$

$$\Rightarrow x^2 - 4x + 68 - 100 = 0$$

$$\Rightarrow x^2 - 4x - 32 = 0$$

$$\Rightarrow x^2 - 8x + 4x - 32 = 0$$

$$\Rightarrow x(x - 8) + 4(x - 8) = 0$$

$$\Rightarrow (x - 8)(x + 4) = 0$$

Either $x - 8 = 0$, then $x = 8$

or $x + 4 = 0$, then $x = -6$

$$\therefore x = 8, -4$$

29. GIVEN: Their steps measure 80 cm, 85 cm and 90 cm.

We have to find the L.C.M of the measures of their steps i.e. 80 cm, 85 cm, and 90 cm, to calculate the required distance each should walk.

L.C.M of 80 cm, 85 cm, and 90 cm.

$$80 = 2^4 \times 5$$

$$85 = 17 \times 5$$

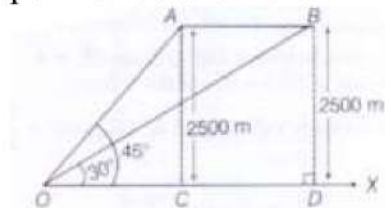
$$90 = 2 \times 3 \times 3 \times 5$$

$$\text{L.C.M of 80, 85 and 90} = 2^4 \times 3 \times 3 \times 5 \times 17$$

$$\text{L.C.M of 80, 85 and 90} = 12240 \text{ cm}$$

Hence, the minimum distance each should walk so that all can cover the same distance in complete steps is 12240 cm.

30. Let OX be the horizontal ground; A and B be the two positions of the plane and O be the point of observation.



Here, $AC = BD = 2500 \text{ m}$,

$$\angle AOC = 45^\circ \text{ and } \angle BOD = 30^\circ$$

$$\text{In right angled } \triangle OCA, \cot 45^\circ = \frac{B}{P} = \frac{OC}{AC}$$



$$\Rightarrow 1 = \frac{OC}{AC}$$

$$\Rightarrow OC = AC = 2500 \text{ m}$$

In right angled $\triangle ODB$, $\cot 30^\circ = \frac{B}{P} = \frac{OD}{BD}$

$$\Rightarrow \sqrt{3} = \frac{OD}{2500}$$

$$\Rightarrow OD = 2500\sqrt{3} \text{ m}$$

Now, $CD = OD - OC = 2500\sqrt{3} - 2500$

$$\Rightarrow CD = 2500(1.732 - 1)$$

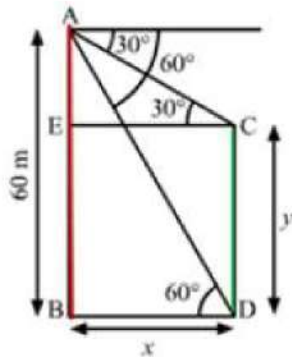
$$\Rightarrow CD = 2500(0.732)$$

$$\Rightarrow CD = 1830 \text{ m}$$

Thus, distance covered by plane in 15 s is 1830 m.

$$\therefore \text{Speed of the plane} = \frac{\text{Distance}}{\text{Time}} = \frac{1830}{15} \times \frac{60 \times 60}{1000} = 439.2 \text{ km/h}$$

OR



Given that AB is a building that is 60m high.

Let $BD = CE = x$ and $BE = y$

$$\Rightarrow AE = AB - BE = 60 - y$$

In $\triangle ABD$,

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \text{ m}$$

In $\triangle AEC$,

$$\tan 30^\circ = \frac{AE}{EC} = \frac{60 - y}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{60 - y}{60/\sqrt{3}}$$

$$\Rightarrow y = 40 \text{ m}$$

The difference between of the building and the lamp post, $AE = 60 - y = 60 - 40 = 20 \text{ m}$

31.

Class	frequency	cummulative frequency
-------	-----------	-----------------------

0-10	2	2
10-20	5	7
20-30	x	7+x
30-40	12	19+x
40-50	17	36+x
50-60	20	56+x
60-70	y	56+x+y
70-80	9	65+x+y
80-90	7	72+x+y
90-100	4	76+x+y

Given, total frequency = 100

So, $76 + x + y = 100$

Or, $x + y = 100 - 76$

Or, $x + y = 24 \dots\dots(1)$

Median class = 50 - 60

$l = 50$; $\frac{n}{2} = 50$; $cf = 36 + x$; $h = 10$; $f = 20$; Median = 52.5

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$$

$$\text{Or, } 52.5 = 50 + \left[\frac{50 - (36 + x)}{20} \right] \times 10$$

$$\text{Or, } 52.5 = 50 + 25 - 18 - \frac{x}{2}$$

$$\text{Or, } 52.5 = 57 - \frac{x}{2}$$

$$\therefore \frac{x}{2} = 57 - 52.5$$

$$\Rightarrow \frac{x}{2} = 4.5$$

Or, $x = 9$

Substituting $x=9$ in equation (1), we get :-

$$9 + y = 24$$

$$\text{Or, } y = 24 - 9$$

$$\text{Or, } y = 15$$

Hence, $x = 9$ & $y = 15$

Section D

32. Let the length and breadth of the rectangular park be 'L' and 'B' respectively.

Case 1-

If the length and the breadth of the park is decreased by 2 m respectively, its area is decreased by 196 square meter.

Therefore, the area will be = $(L \times B) - 196$

$$(L - 2)(B - 2) = L \times B - 196$$

$$LB - 2B - 2L + 4 = LB - 196$$

$$- 2B - 2L = - 196 - 4$$

$$- 2B - 2L = - 200 \dots(1)$$

Case 2-

If the length of the park is increased by 3 m and breadth is increased by 2 m, then its area is increased by 246 square meter.

Therefore, the area will be = $(L \times B) + 246$

$$(L + 3)(B + 2) = L \times B + 246$$

$$LB + 3B + 2L + 6 = LB + 246$$

$$3B + 2L = 246 - 6$$

$$3B + 2L = 240 \dots(2)$$

Adding equation (1) and (2), we get, $B = 40$

Substituting the value $B = 40$ in equation (2), we get

$$3B + 2L = 240$$

$$3 \times 40 + 2L = 240$$

$$120 + 2L = 240$$

$$2L = 240 - 120$$

$$2L = 120$$

$$L = 60$$

So, the length and breadth of the park is 60 m and 40 m respectively.

OR

Suppose the digits at units and tens place of the given number be x and y respectively.

\therefore the number is $10y + x$.

The number is 4 more than 6 times the sum of the two digits.

$$\therefore 10y + x = 6(x + y) + 4$$

$$\Rightarrow 10y + x = 6x + 6y + 4$$

$$\Rightarrow 6x + 6y - 10y - x = - 4$$

$$\Rightarrow 5x - 4y = - 4 \dots(i)$$

After interchanging the digits, the number becomes $10x + y$.

If 18 is subtracted from the number, the digits are reversed. Thus, we have

$$(10y + x) - 18 = 10x + y$$

$$\Rightarrow 10x + y - 10y - x = - 18$$

$$\Rightarrow 9x - 9y = - 18$$

$$\Rightarrow 9(x - y) = - 18$$

$$\Rightarrow x - y = \frac{-18}{9}$$

$$\Rightarrow x - y = - 2 \dots(ii)$$

So, we have the systems of equations

$$5x - 4y = - 4,$$

$$x - y = - 2$$

Here x and y are unknowns. We have to solve the above systems of equations for x and y .

Multiplying the second equation by 5 and then subtracting from the first, we have

$$(5x - 4y) - (5x - 5y) = -4 - (-2 \times 5)$$

$$\Rightarrow 5x - 4y - 5x + 5y = - 4 + 10$$

$$\Rightarrow y = 6$$

Substituting the value of y in the second equation, we have

$$x - 6 = -2$$

$$\Rightarrow x = 6 - 2$$

$$\Rightarrow x = 4$$

Hence, the number is $10 \times 6 + 4 = 64$

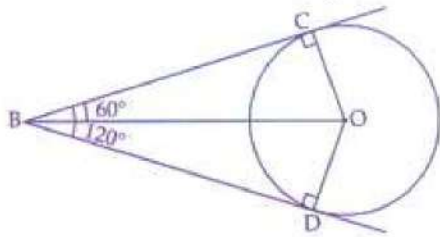
33. According to the question, we are given that from an external point B of a circle with centre ' O ', two tangents BC , BD are drawn such that $\angle DBC = 120^\circ$, we have to prove that $BC + BD = BO$, i.e., $BO = 2BC$.

Given: A circle with centre O .

Tangents BC and BD are drawn from an external point B such that $\angle DBC = 120^\circ$

To prove: $BC + BD = BO$, i.e., $BO = 2BC$

Construction: Join OB , OC and OD .



Proof: In $\triangle OBC$ and $\triangle OBD$, we have

$OB = OB$ [Common]

$OC = OD$ [Radii of same circle]

$BC = BD$ [Tangents from an external point are equal in length] ... (i)

$\therefore \triangle OBC \cong \triangle OBD$ [By SSS criterion of congruence]

$\Rightarrow \angle OBC = \angle OBD$ (CPCT)

$$\therefore \angle OBC = \frac{1}{2} \angle DBC = \frac{1}{2} \times 120^\circ [\because \angle CBD = 120^\circ \text{ given}]$$

$$\Rightarrow \angle OBC = 60^\circ$$

OC and BC are radius and tangent respectively at contact point C .

Hence, $\angle OCB = 90^\circ$

Now, in right angle $\triangle OCB$, $\angle OBC = 60^\circ$

$$\therefore \cos 60^\circ = \frac{BC}{BO}$$

$$\Rightarrow \frac{1}{2} = \frac{BC}{BO}$$

$$\Rightarrow OB = 2BC$$

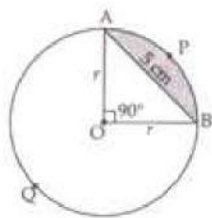
$$\Rightarrow OB = BC + BC$$

$$\Rightarrow OB = BC + BD [\because BC = BD \text{ from (i)}]$$

Hence, proved.

34. Chord $AB = 5$ cm divides the circle into two segments minor segment APB and major segment AQB . We have to find out the difference in area of major and minor segment. Here, we are given that $\theta = 90^\circ$

$$\text{Area of } \triangle OAB = \frac{1}{2} \text{Base} \times \text{Altitude} = \frac{1}{2}r \times r = \frac{1}{2}r^2$$



Area of minor segment APB

$$= \frac{\pi r^2 \theta}{360^\circ} - \text{Area of } \triangle AOB$$

$$= \frac{\pi r^2 90^\circ}{360^\circ} - \frac{1}{2} r^2$$

$$\Rightarrow \text{Area of minor segment} = \left(\frac{\pi r^2}{4} - \frac{r^2}{2} \right) \dots \text{(i)}$$

Area of major segment AQB = Area of circle – Area of minor segment

$$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2} \right]$$

$$\Rightarrow \text{Area of major segment AQB} = \left[\frac{3}{4} \pi r^2 + \frac{r^2}{2} \right] \dots \text{(ii)}$$

Difference between areas of major and minor segment

$$= \left(\frac{3}{4} \pi r^2 + \frac{r^2}{2} \right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2} \right)$$

$$= \frac{3}{4} \pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$$

$$\Rightarrow \text{Required area} = \frac{2}{4} \pi r^2 + r^2 = \frac{1}{2} \pi r^2 + r^2$$

In right $\triangle OAB$,

$$r^2 + r^2 = AB^2$$

$$\Rightarrow 2r^2 = 5^2$$

$$\Rightarrow r^2 = \frac{25}{2}$$

$$\text{Therefore, required area} = \left[\frac{1}{2} \pi \times \frac{25}{2} + \frac{25}{2} \right] = \left[\frac{25}{4} \pi + \frac{25}{2} \right] \text{cm}^2$$

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have $a = 35$, $b = 53$ and $c = 66$.

$$\therefore s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77\text{cm}$$

Let Δ_1 be the area of the first triangle. Then,

$$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$$

$$\Rightarrow \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11} =$$

$$\sqrt{7^2 \times 11^2 \times 6^2 \times 2^2} = 7 \times 11 \times 6 \times 2 = 924\text{cm}^2 \dots(i)$$

For the second triangle, we have $a = 33$, $b = 56$, $c = 65$

$$\therefore s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77\text{cm}$$

Let Δ_2 be the area of the second triangle. Then,

$$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$$

$$\Rightarrow \Delta_2 = \sqrt{77 \times 44 \times 21 \times 12} =$$

$$\sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4} = \sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$$

$$\Rightarrow \Delta_2 = 7 \times 11 \times 4 \times 3 = 924\text{cm}^2$$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$$\Rightarrow \pi r^2 = \Delta_1 + \Delta_2$$

$$\Rightarrow \pi r^2 = 924 + 924$$

$$\Rightarrow \frac{22}{7} \times r^2 = 1848$$

$$\Rightarrow r^2 = 1848 \times \frac{7}{22} = 3 \times 4 \times 7 \times 7 \Rightarrow r = \sqrt{3 \times 2^2 \times 7^2} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}\text{cm}$$

35. Total number of cards in a deck of playing cards = 52

\therefore Number of all possible outcomes = 52

$$\text{Probability of the event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

i. Let E be the event that the card drawn is a king or jack.

Then, the number of outcomes favourable to E is $4 + 4 = 8$

$$\text{So, } P(E) = P(\text{a king or Jack}) = \frac{8}{52} = \frac{2}{13}$$

ii. Let E be the event that the card drawn is a non-ace.

Then, the number of outcomes favourable to E is $52 - 4 = 48$.

$$\text{So, } P(E) = P(\text{a non-ace}) = \frac{48}{52} = \frac{12}{13}$$

iii. Let E be the event that the card drawn is a red card.

Then, the number of outcomes favourable to E is $13 + 13 = 26$.



$$\text{So, } P(E) = P(\text{a red-card}) = \frac{26}{52} = \frac{1}{2}$$

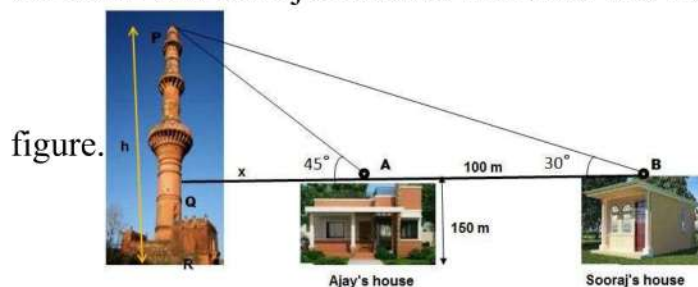
- iv. Let E be the event that the card drawn is neither a king nor a queen. Then, the number of outcomes favourable to E is $52 - (4 + 4) = 44$.

$$\text{So, } P(E) = P(\text{neither a king nor a queen}) = \frac{44}{52} = \frac{11}{13}$$

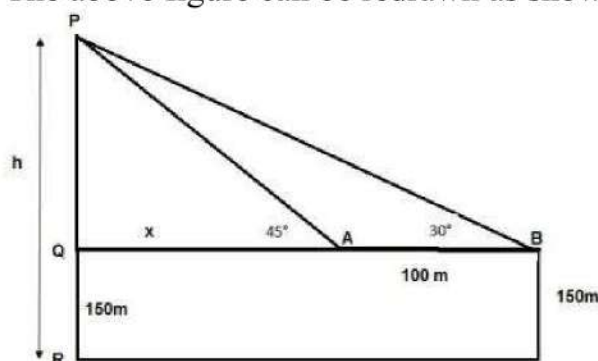
Section E

36. Read the text carefully and answer the questions:

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of Ajay's house to the tower and Sooraj's house to the tower are 45° and 30° respectively as shown in the



- (i) The above figure can be redrawn as shown below:



$$\text{Let } PQ = y$$

In $\triangle PQA$,

$$\tan 45 = \frac{PQ}{QA} = \frac{y}{x}$$

$$1 = \frac{y}{x}$$

$$x = y \dots (i)$$

In $\triangle PQB$,

$$\tan 30 = \frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$$

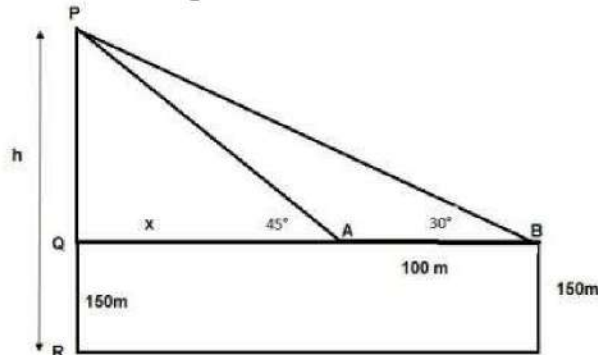
$$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$$

$$x\sqrt{3} = x + 100$$

$$x = \frac{100}{\sqrt{3}-1} = 136.61 \text{ m}$$

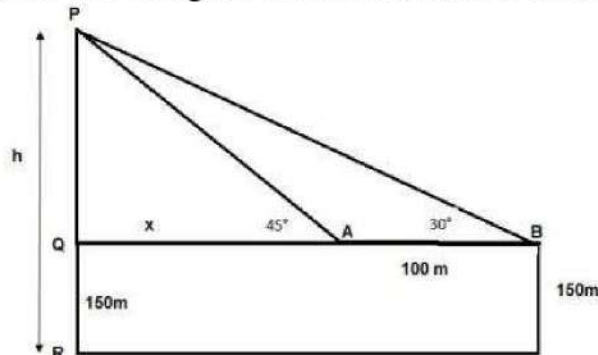
From the figure, height of tower $h = PQ + QR$
 $= x + 150 = 136.61 + 150$
 $h = 286.61 \text{ m}$

(ii) The above figure can be redrawn as shown below:



Distance of Sooraj's house from tower $= QA + AB$
 $= x + 100 = 136.61 + 100 = 236.61 \text{ m}$

(iii) The above figure can be redrawn as shown below:



Distance between top of tower and Top of Sooraj's house is PB
 In $\triangle PQB$

$$\sin 30^\circ = \frac{PQ}{PB}$$

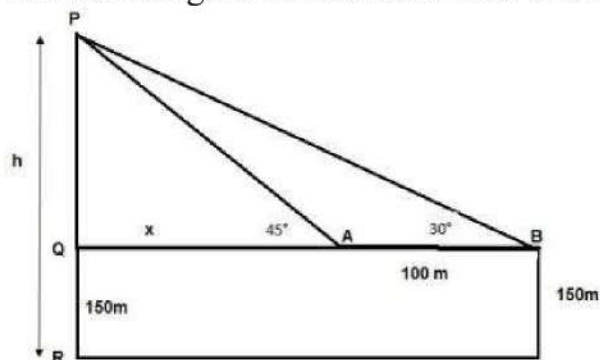
$$\Rightarrow PB = \frac{PQ}{\sin 30^\circ}$$

$$\Rightarrow PB = \frac{y}{\frac{1}{2}} = 2 \times 136.61$$

$$\Rightarrow PB = 273.20 \text{ m}$$

OR

The above figure can be redrawn as shown below:



Distance between top of the tower and top of Ajay's house is PA

In $\triangle PQA$

$$\sin 45^\circ = \frac{PQ}{PA}$$

$$\Rightarrow PA = \frac{PQ}{\sin 45^\circ}$$

$$\Rightarrow PA = \frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$$

$$\Rightarrow PA = 193.20 \text{ m}$$

37. Read the text carefully and answer the questions:

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) Child's Day wise are,

$$\frac{5}{1 \text{ coin}}, \frac{10}{2 \text{ coins}}, \frac{15}{3 \text{ coins}}, \frac{20}{4 \text{ coins}}, \frac{25}{5 \text{ coins}}, \dots \text{ to } \frac{n \text{ days}}{n \text{ coins}}$$

We can have at most 190 coins

i.e., $1 + 2 + 3 + 4 + 5 + \dots$ to n term = 190

$$\Rightarrow \frac{n}{2} [2 \times 1 + (n - 1)1] = 190$$

$$\Rightarrow n(n + 1) = 380 \Rightarrow n^2 + n - 380 = 0$$

$$\Rightarrow (n + 20)(n - 19) = 0 \Rightarrow (n + 20)(n - 19) = 0$$

$$\Rightarrow n = -20 \text{ or } n = 19 \Rightarrow n = -20 \text{ or } n = 19$$

But number of coins cannot be negative

$$\therefore n = 19 \text{ (rejecting } n = -20)$$

So, number of days = 19

(ii) Total money she saved = $5 + 10 + 15 + 20 + \dots = 5 + 10 + 15 + 20 + \dots$ upto 19 terms

$$\begin{aligned} &= \frac{19}{2}[2 \times 5 + (19 - 1)5] \\ &= \frac{19}{2}[100] = \frac{1900}{2} = 950 \end{aligned}$$

and total money she saved = ₹950

(iii) Money saved in 10 days

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n - 1)d] \\ \Rightarrow S_{10} &= \frac{10}{2}[2 \times 5 + (10 - 1) \times 5] \\ \Rightarrow S_{10} &= 5[10 + 45] \\ \Rightarrow S_{10} &= 275 \end{aligned}$$

Money saved in 10 days = ₹275

OR

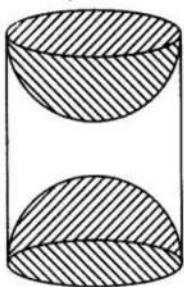
Number of coins in piggy bank on 15th day

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n - 1)d] \\ \Rightarrow S_{15} &= \frac{15}{2}[2 \times 5 + (15 - 1) \times 5] \\ \Rightarrow S_{15} &= \frac{15}{2}[2 + 14] \\ \Rightarrow S_{15} &= 120 \end{aligned}$$

So, there are 120 coins on 15th day.

38. Read the text carefully and answer the questions:

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Given:

Length of rectangle = 12 cm

Width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

Height of cylinder = 10 cm

Diameter of base = 7 cm

⇒ Radius of base = 3.5 cm

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5^2 \times 10 = 385 \text{ cm}^3$$

(ii) Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

⇒ radius of base = 3.5 cm

Volume of wood scooped out = 2 × volume of hemisphere

$$\Rightarrow \text{Volume of wood scooped-out} = 2 \times \frac{2}{3} \times \pi \times r^3$$

$$\Rightarrow \text{Volume of wood scooped out} = \frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.66 \text{ cm}^3$$

(iii) Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

⇒ radius of base = 3.5 cm

Total surface area of the article

$$= 2\pi(3.5)(10) + 2 \left[2\pi(3.5)^2 \right]$$

$$= 70\pi + 49\pi = 119\pi$$

$$= 119 \times \frac{22}{7} = 17 \times 22$$

$$= 374 \text{ cm}^2$$

OR

Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

⇒ radius of base = 3.5 cm

T.S.A of cylinder = $2\pi r(r + h)$

$$\Rightarrow \text{T.S.A of cylinder} = 2 \times \frac{22}{7} \times 3.5(3.5 + 10) = 99 \text{ cm}^2$$

